NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 3045

ANALOGY BETWEEN MASS AND HEAT TRANSFER WITH TURBULENT FLOW

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Washington
October 1953

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SUMMARY

An analysis of combined heat and mass transfer from a flat plate has been made in terms of Prandtl's simplified physical concept of the turbulent boundary layer. The results of the analysis show that for conditions of reasonably small heat and mass transfer, the ratio of the mass- and heat-transfer coefficients is dependent on the Reynolds number of the boundary layer, the Prandtl number of the medium of diffusion, and the Schmidt number of the diffusing fluid in the medium of diffusion. For the particular case of water evaporating into air, the ratio of mass-transfer coefficient to heat-transfer coefficient is found to be slightly greater than unity.

INTRODUCTION

In recent years a considerable number of problems have arisen which involve the calculation of simultaneous mass and heat transfer at high speeds. The evaporative cooling of surfaces by air streams at high speed and at large Reynolds numbers is of considerable interest (ref. 1).

Although research on both mass and heat transfer has been conducted for many years and a number of analyses made, the status of the problem is such that it would seem desirable to present an analysis of the processes based on the simplified physical picture of the turbulent boundary layer and its laminar sublayer as originally conceived by Prandtl.

The analysis presented herein is based on Donaldson's modification (ref. 2) of the Prandtl boundary-layer concept, which permits the calculation of laminar-sublayer characteristics with a temperature variation through the boundary layer, and the use of Reynolds analogy in the turbulent region outside the sublayer. Such an analysis is, in general, subject to certain restrictive conditions since it must be assumed that the momentum boundary layer is unaffected by the mass- and heat-transfer processes. The application of the analysis is therefore limited to the conditions of reasonably small temperature and partial pressure gradients across the boundary layer, small partial pressure of the material being transported in relation to the partial pressure of the medium of diffusion, and a Prandtl number of the medium of diffusion of order unity. Most gases, however, meet this requirement on Prandtl number.

The requirements on the partial pressure of the fluids and the temperature gradient across the boundary layer are realized in at least several practical problems in aeronautics. In particular in the thermal de-icing of aircraft components, all the usual assumptions are met. 3004

The purpose of this paper is to present an analysis of the mass- and heat-transfer process in terms of a simplified physical picture of the turbulent boundary layer subject to the assumptions previously described.

SYMBOLS

The following symbols are used in this report:

C_f	skin-friction coefficient
c_p	specific heat of medium of diffusion at constant pressure, Btu/(lb)(°F)
D	diffusion coefficient, ft^2/sec
g	acceleration due to gravity, ft/sec^2
k_e	mass-transfer coefficient or Stanton number for mass transfer
k_h	heat-transfer coefficient or Stanton number for heat transfer
m,k	constants
n	exponent of boundary-layer power-law profile, $\frac{u}{u_0} = \left(\frac{y}{\delta}\right)^{\frac{1}{n}}$
Pr	Prandtl number, $\frac{\mu c_p g}{\kappa}$
Q	heat transfer per unit area per unit time, Btu/(sec)(ft^2)
Re_L	Reynolds number, $\frac{\rho_L u_L \delta_L}{\mu_L}$
Re_x	Reynolds number, $\frac{\rho_0 u_0 x}{\mu_0}$

Re_δ Reynolds number, $\frac{\rho_0 u_0 \delta}{\mu_0}$

r definitive ratio of total shear stress to laminar shear stress in boundary layer

Sc Schmidt number, $\frac{\mu}{\rho D}$

T absolute temperature, $^{\circ}R$

u velocity, ft/sec

W weight of medium diffusing from surface per unit area per unit time, lb/(ft^2)(sec)

x distance along surface from stagnation point, ft

y distance normal to surface, ft

δ boundary-layer thickness, ft

δ_L laminar-sublayer thickness, ft

η concentration, weight of diffusing material per unit weight of medium of diffusion, lb/lb

κ thermal conductivity of medium of diffusion, Btu/(ft)(sec)($^{\circ}F$)

μ viscosity, slug/(ft)(sec)

ν kinematic coefficient of viscosity, μ/ρ, ft^2/sec

ρ mass density of medium of diffusion, slug/cu ft

σ mutual diffusivity, $\frac{\text{lb (medium of diffusion)}}{\text{(ft)(sec)}}$

Subscripts:

0 free-stream conditions

adw adiabatic wall

L conditions at edge of laminar sublayer

w wall

ANALYSIS

Flow Over Flat Plate at Low Speeds

The study of reference 2 relates the local skin-friction coefficient and the local Reynolds number based on boundary-layer thickness for the case of turbulent flow over a flat plate. The analysis described therein is based on the simplified physical concept of the boundary layer shown in figure 1. The boundary layer is assumed to be sharply divided into a turbulent region having a power-law velocity profile and a laminar region having a constant shear stress. The thickness of the laminar sublayer is given by the intersection of the turbulent-power-law velocity profile and the velocity profile of the laminar sublayer. This conception of the boundary layer is quite similar to that originally presented by Prandtl, which assumed a turbulent region and a laminar sublayer wherein the velocity increased linearly with distance from the surface.

The relations developed in reference 2 are in good agreement with the experimental evidence presented; hence, it may be assumed that for purposes of analysis the simplified physical picture of the boundary layer assumed by Donaldson (ref. 2) is justified.

The general heat-transfer equation based on the temperature potential across the boundary layer, which is given in reference 3 is, in the notation of this report,

$$Q = k_h \rho_0 g u_0 c_{p,0} \left(T_W - T_0\right) \tag{1}$$

Similarly, the mass-transfer equation based on the concentration potential across the boundary layer may be written as

$$W = k_e \rho_0 g u_0 \left(\eta_W - \eta_0\right) \tag{2}$$

The heat transferred at the surface and throughout the laminar sublayer results purely from conduction and may be written in the usual fashion as

$$Q = -\kappa_W \left.\frac{\partial T}{\partial y}\right]_W = -\kappa_L \left.\frac{\partial T}{\partial y}\right]_L \tag{3}$$

Similarly, the mass transfer at the wall may be written as

$$W = -\sigma_W \left.\frac{\partial \eta}{\partial y}\right]_W = -\sigma_L \left.\frac{\partial \eta}{\partial y}\right]_L \tag{4}$$

where σ is the mutual diffusivity based on the diffusion characteristics of the substances involved in the process. In practical engineering terms, σ is usually given as ρDg where D is the diffusion coefficient for the particular process under consideration.

The process of diffusion from the surface with its accompanying finite velocity of the escaping molecules is assumed not to affect the momentum boundary layer or heat transfer of the working medium. This assumption seems reasonable since the diffusing vapor represents only a small fraction of the medium of diffusion.

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The use of equation (4) (Fick's law) in this form implies that the partial pressure of the substance being transported is small compared with the pressure of the working medium. In addition, the mutual diffusivity σ is defined for a field of uniform temperature; if large temperature differences exist, thermal diffusion will be superimposed over the mechanical diffusion and the simple form of Fick's law will no longer apply. The mass-transfer analysis contained herein is therefore limited to cases where the temperature differences are small with respect to the absolute temperature of the medium of diffusion and where the partial pressures of the diffusing material are small compared with partial pressure of the working medium.

It is assumed that the temperature and concentration gradients at the edge of the laminar sublayer are given by

$$\left.\frac{\partial T}{\partial y}\right]_L \cong \frac{T_L - T_w}{\delta_L} \tag{5}$$

$$\left.\frac{\partial \eta}{\partial y}\right]_L \cong \frac{\eta_L - \eta_w}{\delta_L} \tag{6}$$

This assumption necessarily implies that the thickness of the laminar sublayer is identical for both heat and mass transfer. There is, however, no a priori reason to expect that the physical thickness of the sublayer would differ for the two processes. Donaldson (ref. 2) has shown that for the incompressible case with zero heat transfer, the laminar-sublayer thickness is dependent on a characteristic Reynolds number based on the velocity at the edge of the sublayer, the laminar-sublayer thickness, and the viscosity. Such a characteristic Reynolds number may be assumed applicable to heat and mass transfer provided the transfer rates are low. From equations (1), (3), and (5), and (2), (4), and (6), the following relations may be obtained:

$$k_h \rho_0 g u_0 c_{p,0} \left(T_w - T_0\right) = \kappa_L \frac{T_w - T_L}{\delta_L} \tag{7}$$

$$k_e \rho_0 g u_0 \ (\eta_w - \eta_0) = \sigma_L \frac{\eta_w - \eta_L}{\delta_L} \tag{8}$$

Rearranging terms and multiplying the left-hand side of equation (7) by μ_0/μ_0, κ_0/κ_0, and δ/δ yield the following

$$k_h \frac{\rho_0 u_0 \delta}{\mu_0} \frac{c_{p,0} g \mu_0}{\kappa_0} \frac{\kappa_0}{\kappa_L} \frac{\delta_L}{\delta} = \frac{T_w - T_L}{T_w - T_0}$$

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but

$$\frac{c_{p,0} g \mu_0}{\kappa_0} = Pr_0$$

and

$$\frac{\rho_0 u_0 \delta}{\mu_0} = Re_\delta$$

$$k_h \ Re_\delta \ Pr_0 \frac{\kappa_0}{\kappa_L} \frac{\delta_L}{\delta} = \frac{T_w - T_L}{T_w - T_0} \tag{9}$$

A similar result may be obtained for equation (8) by rearranging terms, multiplying the left-hand side by μ_0/μ_0, δ/δ, and μ_L/μ_L and substituting $\sigma_L = \rho_L D_L g$.

$$k_e \frac{\rho_0 u_0 \delta}{\mu_0} \frac{\mu_L}{\rho_L D_L} \frac{\mu_0}{\mu_L} \frac{\delta_L}{\delta} = \frac{\eta_w - \eta_L}{\eta_w - \eta_0}$$

but

$$\frac{\mu_L}{\rho_L D_L} = Sc_L$$

therefore

$$k_e \ Re_\delta \ Sc_L \frac{\mu_0}{\mu_L} \frac{\delta_L}{\delta} = \frac{\eta_w - \eta_L}{\eta_w - \eta_0} \tag{10}$$

It is interesting to note the similarity between equations (9) and (10) with the exception that in equation (9) the Prandtl number is based on stream values and the Schmidt number in equation (10) is based on values at the edge of the laminar sublayer. In general, however, this is not important since the Prandtl number for air varies only slightly with temperature and the Schmidt number for many combinations of fluids is also only a slight function of temperature.

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The temperature ratio $\frac{T_w - T_L}{T_w - T_0}$ and the concentration ratio $\frac{\eta_w - \eta_L}{\eta_w - \eta_0}$ may be evaluated in terms of the laminar-sublayer velocity u_L and the stream velocity u_0 with the aid of Reynolds analogy. The following derivation is similar to that found in many heat-transfer text books; see, for example, reference 4. It is repeated here to obtain specific relations required by the present analysis.

Consider an area in the turbulent region of the boundary layer as shown in figure 2. Let β be a small mass of fluid which is transported per unit time, per unit area across the plane A-A. Suppose that β penetrates upward through A-A to a region of higher velocity u', lower temperature T', and lower concentration η'. In the steady-state condition (completely developed turbulence), an equal mass β must be transported downward across plane A-A to a region of lower velocity u, higher temperature T, and higher concentration η.

The exchange in momentum across A-A is $\beta u - \beta u'$. This change in momentum must be balanced by a change in the shear stress such that the "virtual turbulent shear stress" τ_t must be equal to $\beta u - \beta u'$.

$$\tau_t = -\beta(u - u') \tag{11}$$

Similarly the heat Q_t transported across A-A must be given by

$$Q_t = +\beta g c_p(T - T') \tag{12}$$

and the mass transported across A-A by

$$W_t = +\beta g(\eta - \eta') \tag{13}$$

To eliminate β, which is unknown, from the equations, equations (12) and (13) are divided by equation (11) as follows:

$$\frac{Q_t}{\tau_t} = - \frac{gc_p(T - T')}{u - u'} \tag{14}$$

$$\frac{W_t}{\tau_t} = - \frac{\eta - \eta'}{u - u'} g \tag{15}$$

From equations (3) to (6), it is found that

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$$Q = \kappa_L \frac{T_w - T_L}{\delta_L} \tag{16}$$

$$W = \rho_L D_L g \frac{\eta_w - \eta_L}{\delta_L} \tag{17}$$

Also the shear stress at the edge of the laminar sublayer is given in reference 2 as

$$\tau_L = \mu_L \frac{u_L}{\delta_L} \tag{18}$$

In accordance with the concept of a boundary layer with a sharp division between the laminar and turbulent regions, it is therefore assumed that heat and mass transfer in the turbulent region result entirely from turbulent interchange, that conductive effects are negligible, and further that in the laminar sublayer heat and mass transfer occur purely by conduction and diffusion, respectively.

Therefore at the edge of the laminar sublayer, $Q_t = Q$, $W_t = W$, and $\tau = \tau_t$; and the following results may be obtained from equations (14) to (18) since $\frac{T - T'}{u - u'} = \frac{T_L - T_0}{u_L - u_0}$:

$$\frac{\kappa_L \dfrac{T_w - T_L}{\delta_L}}{\mu_L \dfrac{u_L}{\delta_L}} = - \frac{gc_{p,L}(T_L - T_0)}{u_L - u_0}$$

$$\frac{T_w - T_L}{T_L - T_0} = \frac{gc_{p,L}\mu_L}{\kappa_L} \frac{u_L}{u_0 - u_L} = Pr_L \frac{u_L}{u_0 - u_L}$$

$$\frac{\rho_L D_L \dfrac{\eta_w - \eta_L}{\delta_L}}{\mu_L \dfrac{u_L}{\delta_L}} = -\frac{\eta_L - \eta_0}{u_L - u_0}$$

$$\frac{\eta_w - \eta_L}{\eta_L - \eta_0} = \frac{\mu_L}{\rho_L D_L} \frac{u_L}{u_0 - u_L} = Sc_L \frac{u_L}{u_0 - u_L}$$

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Now

$$\frac{T_w - T_L}{T_w - T_0} = \frac{T_w - T_L}{T_L - T_0 + T_w - T_L} = \frac{\dfrac{T_w - T_L}{T_L - T_0}}{1 + \dfrac{T_w - T_L}{T_L - T_0}}$$

$$= \frac{Pr_L \dfrac{u_L}{u_0 - u_L}}{1 + Pr_L \dfrac{u_L}{u_0 - u_L}} = \frac{Pr_L \dfrac{u_L}{u_0}}{1 - \dfrac{u_L}{u_0}(1 - Pr_L)} \tag{19}$$

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Similarly,

$$\frac{\eta_w - \eta_L}{\eta_w - \eta_0} = \frac{Sc_L \dfrac{u_L}{u_0}}{1 - \dfrac{u_L}{u_0}(1 - Sc_L)} \tag{20}$$

Donaldson (ref. 2) has established that

$$\frac{u_L}{u_0} = \left[\frac{n(r - 1)}{k^2} \frac{\nu_L}{u_0 \delta}\right]^{\frac{1}{n+1}} \tag{21}$$

$$\frac{\delta_L}{\delta} = \left[\frac{n(r - 1)}{k^2} \frac{\nu_L}{u_0 \delta}\right]^{\frac{n}{n+1}} \tag{22}$$

where $\frac{n(r - 1)}{k^2}$ is found to represent a characteristic Reynolds number of the laminar sublayer Re_L which has a constant value for a given value of n. A value of $Re_L = 158$ is found in reference 2 by a comparison of the analytical expression for the skin friction with an experimental drag law.

$$C_f = 0.045 \, Re_\delta^{-1/4}$$

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This value of Re_L is in good agreement with the critical Reynolds number of the laminar sublayer as determined by Donaldson (ref. 2) from the results of von Kármán (ref. 5). Equations (21) and (22) may be rewritten as

$$\frac{u_L}{u_0} = \left(\frac{Re_L}{Re_\delta}\right)^{\frac{1}{n+1}} \left(\frac{\nu_L}{\nu_0}\right)^{\frac{1}{n+1}}$$

$$\frac{\delta_L}{\delta} = \left(\frac{Re_L}{Re_\delta}\right)^{\frac{n}{n+1}} \left(\frac{\nu_L}{\nu_0}\right)^{\frac{n}{n+1}}$$

now

$$\frac{\nu_L}{\nu_0} = \frac{\mu_L/\rho_L}{\mu_0/\rho_0} = \frac{\mu_L}{\mu_0} \frac{\rho_0}{\rho_L}$$

but μ_L/μ_0 can be expressed to a close approximation as

$$\frac{\mu_L}{\mu_0} = \left(\frac{T_L}{T_0}\right)^m$$

Since the static pressure is constant throughout the boundary layer,

$$\frac{\rho_0}{\rho_L} = \frac{T_L}{T_0}$$

Therefore,

$$\frac{\nu_L}{\nu_0} = \left(\frac{T_L}{T_0}\right)^{1+m}$$

and

$$\frac{u_L}{u_0} = \left(\frac{Re_L}{Re_\delta}\right)^{\frac{1}{n+1}} \left(\frac{T_L}{T_0}\right)^{\frac{1+m}{n+1}} \tag{23}$$

$$\frac{\delta_L}{\delta} = \left(\frac{Re_L}{Re_\delta}\right)^{\frac{n}{n+1}} \left(\frac{T_L}{T_0}\right)^{\frac{n(1+m)}{n+1}} \tag{24}$$

Solving equations (9) and (10) for k_h and k_e, respectively, and using the relations given by equations (19), (20), (23), and (24) give the following results:

$$k_h = \frac{\frac{Pr_L}{Pr_0} Re_\delta^{-\frac{2}{n+1}}}{\frac{\kappa_0}{\kappa_L}\left(\frac{T_L}{T_0}\right)^{\frac{(n-1)(1+m)}{n+1}} Re_L^{\frac{n-1}{n+1}} \left[1 - \left(\frac{Re_L}{Re_\delta}\right)^{\frac{1}{n+1}} \left(\frac{T_L}{T_0}\right)^{\frac{1+m}{n+1}} (1 - Pr_L)\right]} \tag{25}$$

$$k_e = \frac{Re_\delta^{-\frac{2}{\eta+1}}}{\frac{\mu_0}{\mu_L}\left(\frac{T_L}{T_0}\right)^{\frac{(n-1)(1+m)}{n+1}} Re_L^{\frac{n-1}{n+1}} \left[1 - \left(\frac{Re_L}{Re_\delta}\right)^{\frac{1}{n+1}} \left(\frac{T_L}{T_0}\right)^{\frac{1+m}{1+n}} (1 - Sc_L)\right]} \tag{26}$$

It is interesting to note that for the usual values of $n = 7$, $m = 0.76$, and $Re_L = 158$ equation (25) becomes

$$k_h = \frac{\frac{Pr_L}{Pr_0} \ Re_\delta^{-1/4} \ (0.0225)}{\frac{\kappa_0}{\kappa_L}\left(\frac{T_L}{T_0}\right)^{1.32}\left[1 - 1.883 \ Re_\delta^{-1/8}\left(\frac{T_L}{T_0}\right)^{0.22}(1 - Pr_L)\right]}$$

For the case of low heat transfer, that is, low temperature differential across the boundary layer,

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$$\frac{\kappa_0}{\kappa_L} \cong \frac{T_L}{T_0} \cong \frac{Pr_L}{Pr_0} \cong 1$$

Then

$$k_h = \frac{0.0225 \ Re_\delta^{-1/4}}{1 - 1.883 \ Re_\delta^{-1/8} \ (1 - Pr_0)}$$

For a flat plate, empirical results give

$$Re_\delta = 0.37 \ (Re_x)^{4/5}$$

Therefore

$$k_h = \frac{0.0289 \ (Re_x)^{-1/5}}{1 - 2.13 \ Re_x^{-1/10} \ (1 - Pr_0)}$$

This equation is quite similar to the equation given on page 117 of reference 4 and for air differs only slightly in the value of the constants in the numerator and denominator. Values of k_h calculated from this equation and from the equation given in reference 4 agree within several percent, the latter equation giving slightly lower values. This deviation is well within the experimental error found in most heat-transfer measurements. It can be seen, therefore, that equation (25) reduces essentially to a previously established law for heat transfer on a flat plate for the case where the temperature differential across the boundary layer is small.

The ratio of k_e to k_h as given by equations (25) and (26) is as follows:

$$\frac{k_e}{k_h} = \frac{\frac{Pr_0}{Pr_L}\frac{\kappa_0}{\kappa_L}\frac{\mu_L}{\mu_0}\left[1 - \left(\frac{Re_L}{Re_\delta}\right)^{\frac{1}{n+1}}\left(\frac{T_L}{T_0}\right)^{\frac{1+m}{n+1}}(1 - Pr_L)\right]}{1 - \left(\frac{Re_L}{Re_\delta}\right)^{\frac{1}{n+1}}\left(\frac{T_L}{T_0}\right)^{\frac{1+m}{n+1}}(1 - Sc_L)}$$

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In the previous equation the term $\frac{Pr_0}{Pr_L}\frac{\kappa_0}{\kappa_L}\frac{\mu_0}{\mu_L}$ is obviously equal to $\frac{c_{p,0}}{c_{p,L}}$, and hence the equation may be rewritten as follows:

$$\frac{k_e}{k_h} = \frac{\frac{c_{p,0}}{c_{p,L}}\left[1 - \left(\frac{Re_L}{Re_\delta}\right)^{\frac{1}{n+1}}\left(\frac{T_L}{T_0}\right)^{\frac{1.76}{n+1}}(1 - Pr_L)\right]}{1 - \left(\frac{Re_L}{Re_\delta}\right)^{\frac{1}{n+1}}\left(\frac{T_L}{T_0}\right)^{\frac{1.76}{n+1}}(1 - Sc_L)} \tag{27}$$

For the technically interesting case of water evaporating into air, the property values of both air and water are well known. In addition, the diffusion coefficient D for water vapor diffusing through air is the most reliable yet established.

The limitation of the mass-transfer relation mentioned earlier, that is, low partial pressure of the diffusing material with respect to the pressure of the medium of diffusion, limits the applicability of equation (27) to cases where the wall temperatures are much less than the boiling temperature of water.

In addition, the analysis is valid only for the condition where the ratio T_0/T_w is nearly unity and, hence, T_L/T_0 is also nearly unity. Since in equation (27) T_L/T_0 is raised to a power considerably less than one and since the ratio $c_{p,0}/c_{p,L}$ is only slightly affected by temperature variation, equation (27) may be written to a close approximation as

$$\frac{k_e}{k_h} = \frac{1 - \left(\frac{Re_L}{Re_\delta}\right)^{\frac{1}{n+1}} (1 - Pr_L)}{1 - \left(\frac{Re_L}{Re_\delta}\right)^{\frac{1}{n+1}} (1 - Sc_L)} \tag{28}$$

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In the range of temperatures of practical interest (32° to 100° F), (for air) the Prandtl number is approximately equal to 0.71 and for water diffusing into air the Schmidt number is approximately 0.60. The variation of k_e/k_h with Re_δ is shown in figure 3 for the value of $Re_L = 158$ determined in reference 2 and several values of n. Also shown on the figure are curves calculated at a constant value of n equal to 7 and values of Re_L equal to 100 and 300. The range of Reynolds numbers Re_δ shown correspond to a range of Reynolds numbers based on distance from the leading edge of 3.4×10^5 to 1.91×10^9.

It is apparent from figure 3 that the ratio of the mass-transfer coefficient to the heat-transfer coefficient decreases slightly with increasing Reynolds number. The ratio also increases with increasing n, but both the effect of Reynolds number and n are quite small and it would be difficult to isolate either experimentally. The curves shown for values of Re_L equal to 100, 158, and 300 at a constant value of $n = 7$ indicate that the effect of changes in Re_L is so small as to be negligible.

The independence of the ratio k_e/k_h with critical Reynolds number Re_L shows that the assumption made previously about the constancy of Re_L with or without heat transfer is not critical for purposes of the analysis.

From an over-all standpoint, it would therefore be expected that the ratio k_e/k_h can be taken as approximately 1.05 regardless of whether or not the flow is transitional with values of n of the order of 4 or fully turbulent with values of n equal to or greater than 7.

Although there are few actual experimental data on the evaporation of water into an air stream from a flat surface, it has been found that the surface temperature of bodies wetted by a water film, calculated by assuming k_e equal to k_h, agrees quite well with experimental results (see, for example, ref. 1). An unpublished experiment by Coles and Ruggeri on the mass and heat transfer from an iced flat surface for a wide range of altitudes and Mach numbers shows that the ratio of the mass-transfer coefficient to the heat-transfer coefficient is approximately 0.94. These results were obtained with rough, iced surfaces; consequently only qualitative agreement with the present analysis should be expected.

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It is evident that the analysis used herein lends itself to comparative results of heat and mass transfer but can be used to find absolute values of k_e and k_h only when the value of n is known. For extremely high Reynolds numbers, the value of n may be 10 or greater and this would greatly affect the values obtained from equations (25) and (26).

The calculation of the ratio k_e/k_h for fluids other than water into air can be accomplished easily provided a reliable value of diffusion coefficient is available. It is possible that certain fluids may exhibit properties such that the value of T_I/T_0 could be significant without invalidating the assumptions of the analysis. In such a case, the general form of equation (27) must be used and T_I/T_0 obtained from a knowledge of T_w/T_0 and trial-and-error solutions of equations (19) and (23).

Extension to High-Speed Flow

For high-speed flows wherein the frictional temperature rise in the boundary layer is appreciable, it is usual to write the heat-transfer equation in the following manner

$$Q = k_h \rho_0 g u_0 c_p \left(T_w - T_{adw}\right)$$

The use of the adiabatic wall temperature in place of the stream static temperature has been found to result in a satisfactory correlation of k_h with Reynolds number for both low- and high-speed flows (up to Mach numbers of 2).

The relation given by equation (25) would therefore be expected to hold over the whole speed range of current interest for aircraft icing given in reference 1 (Mach numbers from 0 to 1.5). The relation of equation (26) is unaffected by speed provided the conditions stated previously with regard to the vapor pressures and vapor-pressure gradients in the boundary layer are still fulfilled. For the usual values of interest in icing, these values are well within the acceptable limits. The ratio of k_e/k_h as given by equation (28) will therefore be independent of Mach number for the whole range of flight speeds of current interest.

CONCLUSIONS

An analysis of combined heat and mass transfer from a flat plate has been made in terms of Prandtl's simplified physical concept of the boundary layer. The results of the analysis show that for conditions of reasonably small heat and mass transfer, the ratio of the mass- and heat-transfer coefficients is dependent on the Reynolds number of the boundary layer, the Prandtl number of the working fluid, and the Schmidt number of the diffusing material in the medium of diffusion. For the particular case of water evaporating into air, the ratio of mass-transfer coefficient to heat-transfer coefficient is found to be slightly greater than unity. For the particular case of aircraft icing, it is shown that the results of the analysis are valid up to the maximum Mach number at which icing might occur, that is, 1.5.

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Lewis Flight Propulsion Laboratory
National Advisory Committee for Aeronautics
Cleveland, Ohio, August 12, 1953

REFERENCES

1. Callaghan, Edmund E., and Serafini, John S.: Analytical Investigation of Icing Limit for Diamond Shaped Airfoil in Transonic and Supersonic Flow. NACA TN 2861, 1953.

2. Donaldson, Coleman duP.: On the Form of the Turbulent Skin-Friction Law and Its Extension to Compressible Flows. NACA TN 2692, 1952.

3. Goldstein, S, ed.: Modern Developments in Fluid Dynamics. Vol. II. Clarendon Press (Oxford), 1938.

4. Eckert, E. R. G.: Introduction to the Transfer of Heat and Mass. McGraw-Hill Book Co., Inc., 1950.

5. von Kármán, Th.: Turbulence and Skin Friction. Jour. Aero. Sci., vol. 1, no. 1, Jan. 1934, pp. 1-20.

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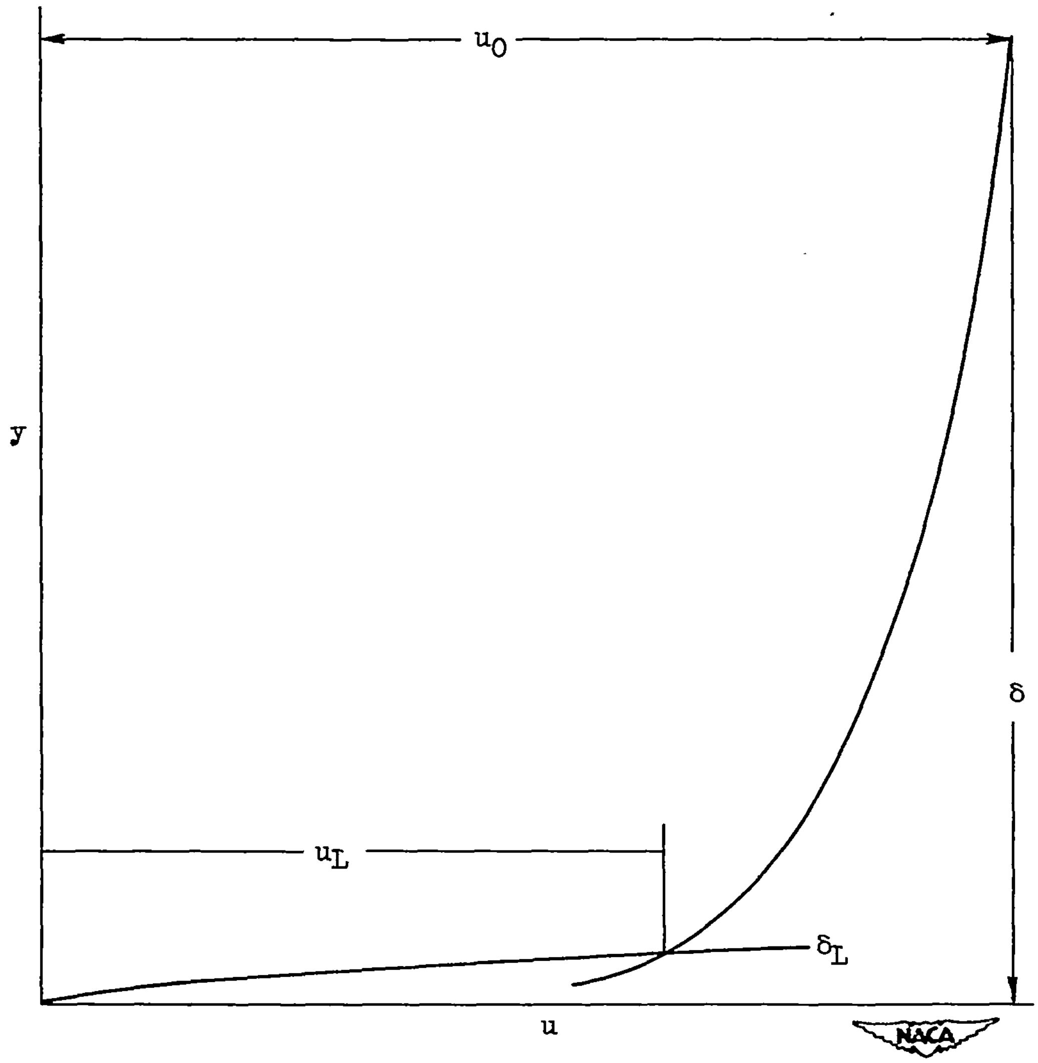

Figure 1. - Velocity profile assumed for analysis.

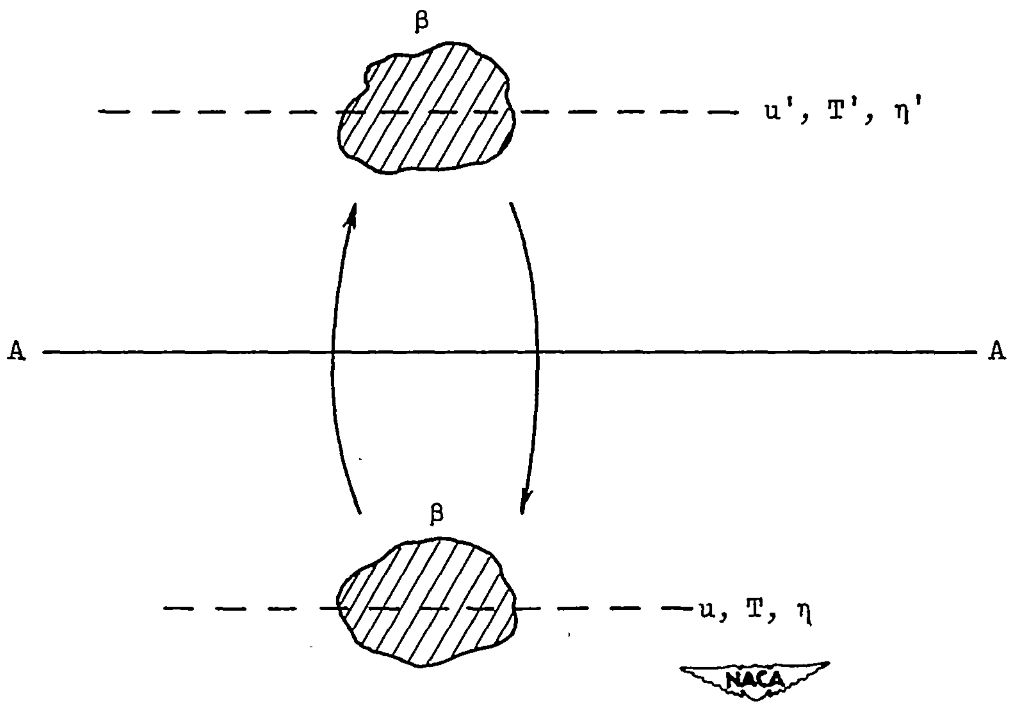

Figure 2. - Simplified picture of turbulent exchange.

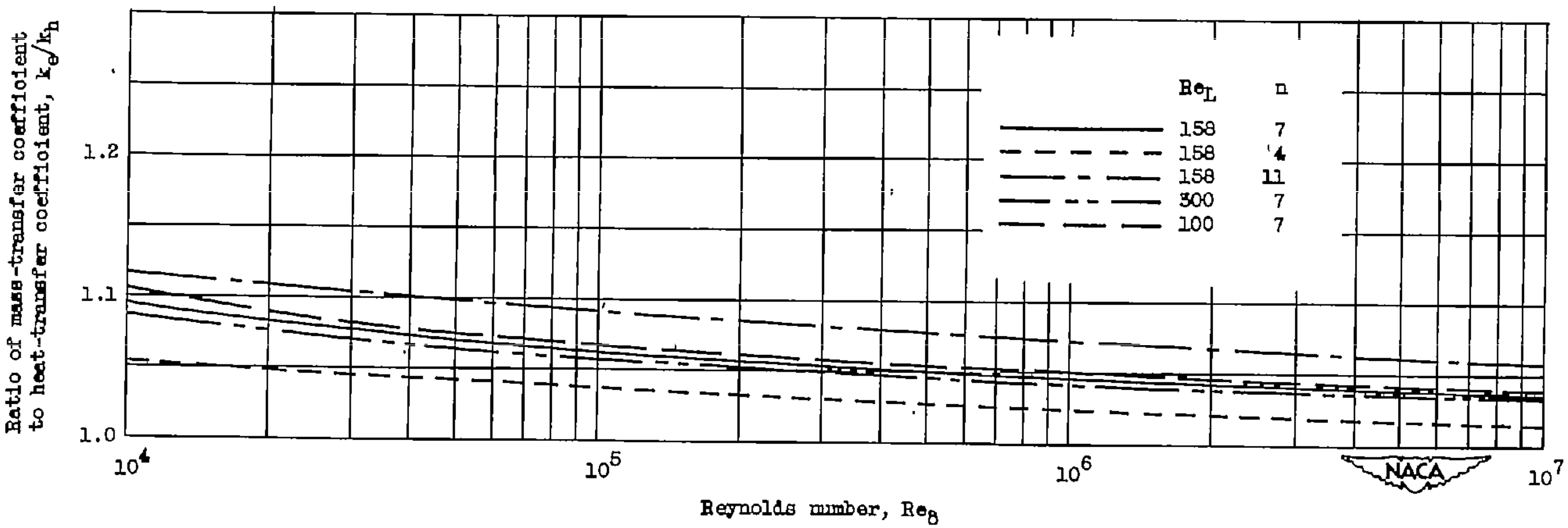

Figure 3. - Variation of ratio of mass-transfer coefficient to heat-transfer coefficient as a function of Reynolds number based on boundary-layer thickness.